USEFULNESS OF SELECTED ANALYTICAL ALGORITHMS FOR CALCULATION OF UNAVAILABILITY OF MARINE POWER PLANT AUXILIARY SYSTEMS

1. Introduction

Fault tree analysis (FTA) method has been widely used as tool for applications of some analytical algorithms for reliability of technical systems analysis since 1961. This method is based on graphical representation of interaction of a number of basic system elements as boolean function. The fundamental concept in this method of analysis is translation physical system like marine power generation and distribution unit into a structured logic diagram, called fault tree, in which certain specified causes lead to one (or more) specified top event of interest. This method can by use to analyse unavailability $Q_0(t)$ of marine power plants [1, 3, 4]. Major topic of this paper is present some of fault tree evaluation methods which can be useful for marine systems.

2. Selected methods of fault tree evaluation for $Q_0(t)$

In this section has been presented selected fault tree evaluation methods for system unavailability $Q_0(t)$ i.e. rare event approximation, Inclusion-Exclusion Expansion, Lower Bound, Upper bound approximation and ERAC algorithm.

2.1. Rare Events Approximation for $Q_0(t)$

Every marine auxiliary installation $XX$ can be presented as series-parallel configuration of $r$ steps [8]. In every serial block are located $s$ parallel paths with $t$ serial connected elements. We can calculate:
\[ TE(XX) = \bigcup_{r=1}^{r(r)} \bigcap_{j=1}^{s(r),s(r)} \bigcup_{j=1}^{E_{jk}} \]  

(2.1)

This equation gives us logical representation of fault tree model for one marine auxiliary installation with non-exclusive and independent primary events (component failures).

Every system is down if and only if one or more of minimal cut sets \( C_1, C_2, \ldots, C_n \) is down. Unavailability of \( k \)-th minimal cut set is given by:

\[ \tilde{Q}_k(t) = \prod_{i \in C_k} q_i(t) \]  

(2.2)

If we ignore the possibility of two or more minimal cut sets being simultaneously down, the system unavailability \( Q_0(t) \) can be thus approximated as the sum of the minimal cut sets unavailabilities \( \tilde{Q}_k(t) \).

\[ \tilde{Q}_{0,rea}(t) = \sum_{k=1}^{n} \tilde{Q}_k(t) \]  

(2.3)

This equation is so-called rare event approximation [10]. It is the most simply method to calculate system unavailability and can be use for \( Q_0(t) < 0.1 \) with results agreeing within 10% of true unavailability. It is possible to observe that, some of marine systems components are part of many different minimal cut sets. And very often during system operation, there is situation where some of elements are critically for system by different cut sets. So, this method of fault tree evaluation methods can have insufficient accuracy and can be use only for some specific conditions.

2.2. Inclusion-Exclusion Expansion and Lower Bound for \( Q_0(t) \)

For fault tree with \( n \) minimal cut sets \( C_i \) it is possible to find exact value of \( Q_0(t) \) for independent events by means of Poincare formula (Inclusion-Exclusion Expansion), given by [10]:

\[ \text{Inclusion-Exclusion Expansion} \]

\[ Q_0(t) = \sum_{k=1}^{n} \tilde{Q}_k(t) - \sum_{k<l}^{n} \tilde{Q}_k(t) \tilde{Q}_l(t) + \sum_{k<l<m}^{n} \tilde{Q}_k(t) \tilde{Q}_l(t) \tilde{Q}_m(t) - \ldots \]

(2.4)

This equation provides a lower bound for \( Q_0(t) \), which is more accurate than the rare event approximation for larger values of \( Q_0(t) \).
\[ Q_0(t) = P[ \bigcup_{i=1}^{n} Q_i(t)] = \sum_{i=1}^{n} Q_i(t) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} Q_i(t)Q_j(t) + \]
\[ \sum_{i=1}^{n-2} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} Q_i(t)Q_j(t)Q_k(t) + \cdots + (-1)^{n-1} Q_i(t)Q_2(t) \cdots Q_n(t) \] (2.4)

It is exact method but formula is difficult to effective calculate for large fault trees, which can be obtain on the basis of marine power plant systems.

Presented before rare event approximation is subpart of this formula, if we consider possibility to simultaneously down one or two different cut sets, we obtain so-called lower bound given by:

\[ \bar{Q}_{0\text{LOW}}(t) = \sum_{i=1}^{n} Q_i(t) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} Q_i(t)Q_j(t) \] (2.5)

2.3. Upper Bound Approximation for \( Q_0(t) \)

The following formula provides an upper bound for \( Q_0(t) \), and is usually a satisfactory approximation to \( Q_0(t) \) [7]. We assume that the system unavailability \( Q_0(t) \) can be approximated as the ip-function of the minimal cut sets unavailabilities \( Q_k(t) \). Let the minimal cut sets of the tree be denoted \( C_1, C_2, \ldots, C_n \). By the assumption of independence of input events, the probability that all input events in the minimal cut set \( C_k \) occur, is given by formula (2.2). If the cut sets were disjoint, then they would be stochastically independent and we have:

\[ \bar{Q}_o(t) = \bar{Q}_{0\text{UBA}}(t) = \prod_{k=1}^{n} \bar{Q}_k(t) = 1 - \prod_{k=1}^{n} [1 - \bar{Q}_k(t)] \] (2.6)

In general, however, the minimal cut sets for marine systems are not disjoint. In this case we always have:

\[ \bar{Q}_o(t) \leq 1 - \prod_{k=1}^{n} [1 - \bar{Q}_k(t)] \] (2.7)

and that in fact \( Q_0(t) \) approximately equals the right hand side of (4.2), at least when the \( q_i(t) \)'s are close to 0. It should be noted that the inequality (4.2) can be also applicable
when the input events in the fault tree are positively dependent (so-called associated) rather than independent.

2.4. Exact Reliability/Availability Calculation

One of alternatives for upper bound approximation is the ERAC algorithm (Exact Reliability/Availability Calculation) which was developed by Aven [1, 5, 6]. The ERAC algorithm is based on a decomposition method by Doulliez and Jamoulle [9], originally designed for transportation networks. A modification of Aven’s approach is used in CARA FaultTree which was use by author of paper for unavailability marine systems simulations.

We assume that fault tree have $n$ independent input events. Let $y = (y_1, y_2, \ldots, y_n)$ denote the random state vector of the input events, where $y_i$ is equal to 1 when $i$-th input event occurs and 0 otherwise. Now, let $A$ denote all the states $y$ of the fault tree such that the TOP event occurs. The probability $Q_o(t)$ of the top event is thus determined by:

$$Q_o(t) = Q_{o,EXACT}(t) = \sum_{y \in A} P[\bar{Y}(t) = \bar{y}]$$

(2.8)

if assume, that:

$$P[Y_i(t) = 1] = q_i(t)$$
$$P[Y_i(t) = 0] = 1 - q_i(t) = p_i(t)$$

then probability, that system is in state given by vector $y$ from set $A$ is given by:

$$P[\bar{Y}(t) = \bar{y}] = \prod_{i=1}^{n} p_i(t)^{1-y_i} q_i(t)^{y_i}$$

(2.9)

The ERAC algorithm and most of its competing algorithms are based on formula (15). The prime objective of all of these algorithms is to determine the set $A$ as efficiently as possible. It is observed that $A$ is always a subset of the vector interval $<0, 1>$. In ERAC the set $A$ is determined by successive partitioning of this interval in so-called acceptable and non-acceptable states.

3. Final conclusions
Upper bound approximation method is giving higher results then obtained with use of ERAC algorithm. Difference of results for two methods is higher for engine rooms working in connected state.

The upper bound approximation for $Q_0(t)$ may in some situations be rather inaccurate and verification of application possibility should be performed, for complex systems like auxiliary marine power plants installations.

Results of calculation $Q_0(t)$, for presented methods, usually can be given as:

$$
\bar{Q}_{e,LOW}(t) < \bar{Q}_{e,EXACT}(t) < \bar{Q}_{e,UBA}(t) < \bar{Q}_{e,REA}(t)
$$

(3.1)

Lower bound and rare even approximation will not be consider in next part of paper as in fact, this methods can provide to high estimation error for marine systems analysis.

Literatura:


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The paper contains presentation selected methods for calculating of unavailability of marine system (rare event approximation, inclusion-exclusion expansion, lower bound, the upper bound approximation and the ERAC algorithm).