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Probabilistic Analysis of Marine Binary Technical Systems Represented by Boolean Models

Key words: logical model, logical operators, fault tree analysis, Boolean rules

Basic relations useful in the reduction of Boolean models of technical systems have been presented. Elementary logical gates used in a coherent fault tree and their probabilistic evaluations have been pointed out. A marine system (sea water cooling system) has been analyzed with the use of the presented methodology.

Analiza probabilistyczna morskich dwustanowych systemów technicznych reprezentowanych modelami boolowskimi

Słowa kluczowe: model logiczny, operatory logiczne, analiza drzewa niezdatności, zasady logiki Boole'a

Przedstawiono podstawowe zależności przydatne przy redukcji boolowskich modeli systemów technicznych. Wyszczególniono podstawowe operatory logiczne wykorzystywane w koherentnych drzewach niezdatności i ich analizie probabilistycznej. Przykładowy system okrętowy (system chłodzenia wodą morską) został przeanalizowany z użyciem przedstawionej metodologii.

Introduction

For a binary model of the system in the form of a fault tree, usually the first step in the dependability analysis is to identify in minimal cut sets of the analysed system. The process of searching for minimal cut sets and path sets is based on the application of Boolean algebra rules to the binary equation which represents a given fault tree model [1, 3, 4].

The identification of minimal cut sets for a given fault tree requires:

1. Conversion of fault tree to equivalent in the form of Boolean formulas set (logical model).
2. Determination of the top event with the use of Boolean algebra by tracing of tree *from bottom to top* or *from top to bottom*.

1. Reduction of logical models

Let E_1, E_2 and E_3 represent any logical events, φ – an empty set, Ω – a full set, and set E' is a complementation of set E. Basic rules of Boolean algebra for these symbols are presented below. These rules are used for the reduction and transformation of Boolean equations, which represent the fault tree model. The most important formulas for fault tree evaluations are:

Commutative Law:

$$E_1 \cap E_2 = E_2 \cap E_1 \tag{1}$$

$$E_1 \cup E_2 = E_2 \cup E_1 \tag{2}$$

Associative Law:

$$E_1 \cap (E_2 \cap E_3) = (E_1 \cap E_2) \cap E_3 \tag{3}$$

$$E_1 \cup (E_2 \cup E_3) = (E_1 \cup E_2) \cup E_3 \tag{4}$$

Idempotent Law:

$$E_1 \cap E_1 = E_1 \tag{5}$$

$$E_1 \cup E_1 = E_1 \tag{6}$$

Law of Absorption:

$$E_1 \cap (E_1 \cup E_2) = E_1 \tag{7}$$

$$E_1 \cup (E_1 \cap E_2) = E_1 \tag{8}$$

Distributive Law:

$$E_1 \cap (E_2 \cup E_3) = (E_1 \cap E_2) \cup (E_1 \cap E_3) \tag{9}$$

$$E_1 \cup (E_2 \cap E_3) = (E_1 \cup E_2) \cap (E_1 \cup E_3) \tag{10}$$

Complementation:

$$E_1 \cap E_1' = \phi \tag{11}$$

$$E_1 \cup E_1' = \Omega \tag{12}$$

De Morgan's Theorem:

$$(E_1 \cap E_2)' = E_1' \cup E_2' \tag{13}$$

$$(E_1 \cup E_2)' = E_1' \cap E_2' \tag{14}$$

Other relations:

$$E_1 \cup (E_1' \cap E_2) = E_1 \cup E_2 \tag{15}$$

$$E_1' \cap (E_1 \cup E_2') = E_1' \cap E_2' \tag{16}$$

2. Logical operators

In the classical fault tree analysis basic kinds of logical gates are used, i.e. union and intersection operators. The structure modelled by means of these operators is always a coherent tree. If for the building of a fault tree also the negation operator is used (or complex gates with internal negation), then the tree may, but does not have to be an incoherent fault tree. These kinds of systems are very rare and will not be analysed in this paper.

The gate *OR* represents the union of input events. If input events are denoted as E_1, E_2,…E_n, and the gate output as *ZP*, the logical representation of *OR* gate operation is given as:

$$ZP = or(E_1, E_2, ... E_n) = E_1 \cup E_2 \cup ... \cup E_n = \bigcup_{i=1}^{i=n} E_i \tag{17}$$

An output event is generated from *OR* gate when there does exist at least one input event. The probability of the output event generation *P*(*ZP*) from *OR* gate with two input events E_1 and E_2 with the probabilities of occurrence, respectively, $P(E_1)$ and $P(E_2)$, according to probabilistic rules is as follows:

$$P(ZP) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = \\ = P(E_1) + P(E_2) - P(E_1)P(E_2 \mid E_1) = \\ = P(E_1) + P(E_2) - P(E_2)P(E_1 \mid E_2) \tag{18}$$

According to the sets theory rules:

- if events E_1 and E_2 are mutually exclusive, which means $P(E_1 \cap E_2) = 0$, then:

$$P(ZP) = P(E_1) + P(E_2) \tag{19}$$

- if input events E_1 and E_2 are independent, which means $P(E_2|E_1) = P(E_2)$, then:

$$P(ZP) = P(E_1) + P(E_2) - P(E_1)P(E_2) \tag{20}$$

- if event E_2 is completely dependent on event E_1, which means $P(E_2|E_1) = 1$, then:

$$P(ZP) = P(E_2) \tag{21}$$

For all cases, if occurrence of two input events in the same time is neglected, the probability can be estimated according to this formula:

$$P(ZP) \cong P(E_1) + P(E_2) \geq P(E_1) + P(E_2) - P(E_1 \cap E_2) \tag{22}$$

For low probabilities of input events (less than 0.1) and for independent input events, the probability of output generation $P(ZP)$ can be estimated with the relative error less than 0.1 with the use of the rare event approximation:

$$P(ZP) \cong P(E_1) + P(E_2) \tag{23}$$

For *OR* gate with n independent input events, the probability of the output event generation is given by Poincare equation:

$$P(ZP) = P(\bigcup_{i=1}^{i=n} E_i) = \sum_{i=1}^{n} P(E_i) - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} P(E_i)P(E_j) + \\ \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} P(E_i)P(E_j)P(E_k) ... + (-1)^{n-1} P(E_1)P(E_2)...P(E_n) \tag{24}$$

If the occurrence of two or more input events in the same time is neglected, this equation will be reduced to the rare event approximation form:

$$P(ZP) = P(\bigcup_{i=1}^{i=n} E_i) = \sum_{i=1}^{n} P(E_i) \tag{25}$$

For completely independent events formula (26) is useful. In practice this equation is used for systems with partly dependent (associated) input events:

$$P(ZP) = P(\bigcup_{i=1}^{i=n} E_i) = 1 - \prod_{i=1}^{i=n} [1 - P(E_i)] = \coprod_{i=1}^{n} P(E_i) \tag{26}$$

The gate *AND* represents an intersection of input events. If input events are denoted as E_1, E_2,…E_n, and the gate output as *ZP*, the logical representation of the *AND* gate operation has this form:

$$ZP = and(E_1, E_2, ...E_n) = E_1 \cap E_2 \cap ... \cap E_n = \bigcap_{i=1}^{i=n} E_i \tag{27}$$

An output event is generated from the *AND* gate when there do exist all of the input events. The probability of output event generation $P(ZP)$ from the *AND* gate with two input events E_1 and E_2 with the probabilities $P(E_1)$ and $P(E_2)$, respectively, according to the probabilistic rules is expressed as:

$$P(ZP) = P(E_1)P(E_2 \mid E_1) = P(E_2)P(E_1 \mid E_2) \tag{28}$$

According to the relevant sets theory rules:

- if input events E_1 and E_2 are independent, $P(E_2 \mid E_1) = P(E_2)$ and $P(E_1 \mid E_2) = P(E_1)$, then:

$$P(ZP) = P(E_1) \cdot P(E_2) \tag{29}$$

- if input events E_1 and E_2 are not independent and $P(E_1) > P(E_2)$, then:

$$P(E_1) \geq P(ZP) > P(E_1) \cdot P(E_2) \tag{30}$$

- if event E_2 is completely dependent on event E_1, which means $P(E_2|E_1) = 1$, then:

$$P(ZP) = P(E_1) \tag{31}$$

For the *AND* gate with n independent input events, the probability of output event generation has this form:

$$P(ZP) = P(\bigcap_{i=1}^{i=n} E_i) = \prod_{i=1}^{i=n} P(E_i) \tag{32}$$

The voting gate represents the logical operation which generates an output event when ther exists at least k out of all n inputs to the gate. This operation is also called K-out-of-N gate. This gate can be represented in the form of *AND* and *OR* gates combination with artificially entered intermediate events [2].

The voting gate is logically a union of all possible k-elements intersections of input events E_1 to E_n. The number of all combinations is :

$$C_k^n = \binom{n}{k} = \frac{n!}{k!(n-k)!} \tag{33}$$

The probability of output event generation $P(ZP)$ through the voting gate with parameters (k, n) and input events $E_1, E_2, \ldots E_n$ is given by this formula:

$$P(ZS) = \sum_{m=k}^{n} \left\{ (-1)^{m+k} \binom{m-1}{k-1} \sum_{\substack{\forall P(E_{Li}) \\ Li \in \{1 \ldots n\}: \\ l1 < \ldots < \ln}} \prod_{j=1}^{m} [P(E_{Li})] \right\} \tag{34}$$

3. Case study

The presented methodology is applicable for time dependent and constant probability models. A case study for selected marine power plant systems installed onboard offshore multi support vessel is shown below. One of these systems: main power plant engines sea water cooling system is presented in Fig. 1.

The description of all components and respective events for the binary model (fault tree) is shown in Table 1. The fault tree is presented in Figure 2. Table 1 also defines the events in the binary model. The analysis was carried out on the basis of the constructed trees taking the calculated failure measures as input data, (moments of failures taken from the real system).

The analysis was done by means of *CARA-FaultTree* computer code from *Sydvest Software*. The calculations basically aimed at the estimation of the unavailability of a selected marine power plant system.

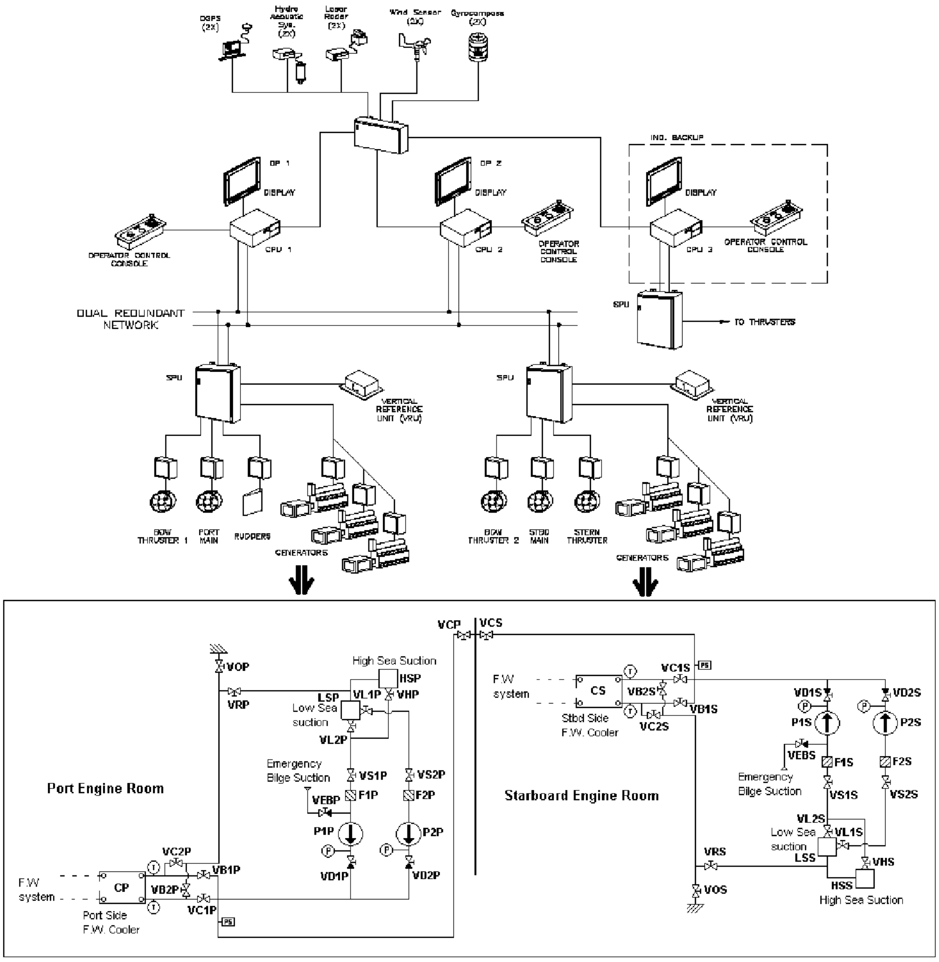

Fig. 1. Diagram of the analyzed sea water cooling system
Rys. 1. Schemat analizowanego systemu chłodzenia wodą morską

Table 1

Description of system components and analyzed events in the system
Opis elementów systemu oraz analizowanych w systemie zdarzeń

Sym-bol	Component name	Type	Event description	Parameter	Value
1	2	3	4	5	6
VL1P	Bottom sea chest valve no 1 Port	On demand	Valve failed in closed position	q [–]	3.0000e–005
VL1S	Bottom sea chest valve no 1 Stbd	On demand	Valve failed in closed position	q [–]	3.0000e–005

Table 1 (continued)

1	2	3	4	5	6
VL2P	Bottom sea chest valve no 2 Port	On demand	Valve failed in closed position	q [–]	3.0000e–005
VL2S	Bottom sea chest valve no 2 Stbd	On demand	Valve failed in closed position	q [–]	3.0000e–005
VHP	High sea chest valve Port	On demand	Valve failed in closed position	q [–]	3.0000e–005
VHS	High sea chest valve Stbd	On demand	Valve failed in closed position	q [–]	3.0000e–005
VOP	Outlet valve Port	On demand	Valve failed in closed position	q [–]	3.0000e–005
VOS	Outlet valve Stbd	On demand	Valve failed in closed position	q [–]	3.0000e–005
VS1P	Suction valve of pump no 1 Port	On demand	Valve failed in closed position	q [–]	3.0000e–005
VS1S	Suction valve of pump no 1 Stbd	On demand	Valve failed in closed position	q [–]	3.0000e–005
VS2P	Suction valve of pump no 2 Port	On demand	Valve failed in closed position	q [–]	3.0000e–005
VS2S	Suction valve of pump no 2 Stbd	On demand	Valve failed in closed position	q [–]	3.0000e–005
VD1P	Delivery valve of pump no 1 Port	On demand	Valve failed in closed position	q [–]	3.0000e–005
VD1S	Delivery valve of pump no 1 Stbd	On demand	Valve failed in closed position	q [–]	3.0000e–005
VD2P	Delivery valve of pump no 2 Port	On demand	Valve failed in closed position	q [–]	3.0000e–005
VD2S	Delivery valve of pump no 2 Stbd	On demand	Valve failed in closed position	q [–]	3.0000e–005
VC1P	Cooler inlet valve Port	On demand	Valve failed in closed position	q [–]	3.0000e–005
VC1S	Cooler inlet valve Stbd	On demand	Valve failed in closed position	q [–]	3.0000e–005
VC2P	Cooler outlet valve Port	On demand	Valve failed in closed position	q [–]	3.0000e–005
VC2S	Cooler outlet valve Stbd	On demand	Valve failed in closed position	q [–]	3.0000e–005
P1P	Sea water pump no 1 Port (active pump)	Non repairable	Failure during starting/running	λ [failure/h]	3.0000e–005
P1S	Sea water pump no 1 Stbd (active pump)	Non repairable	Failure during starting/running	λ [failure/h]	3.0000e–005

Table 1 (continued)

1	2	3	4	5	6
P2P	Sea water pump no 2 Port (standby pump)	On demand	Start on demand failed	q [–]	3.0000e–004
P2S	Sea water pump no 2 Stbd (standby pump)	On demand	Start on demand failed	q [–]	3.0000e–004
F1P	Suction filter no 1 Port	Non repairable	Filter clogged	λ [failure/h]	6.9400e–004
F1S	Suction filter no 1 Stbd	Non repairable	Filter clogged	λ [failure/h]	6.9400e–004
F2P	Suction filter no 2 Port	Non repairable	Filter clogged	λ [failure/h]	6.9400e–004
F2S	Suction filter no 2 Stbd	Non repairable	Filter clogged	λ [failure/h]	6.9400e–004
CP	Central cooler Port	Non repairable	Cooler clogged / seals damaged	λ [failure/h]	1.0000e–006
CS	Central cooler Stbd	Non repairable	Cooler clogged / seals damaged	λ [failure/h]	1.0000e–006

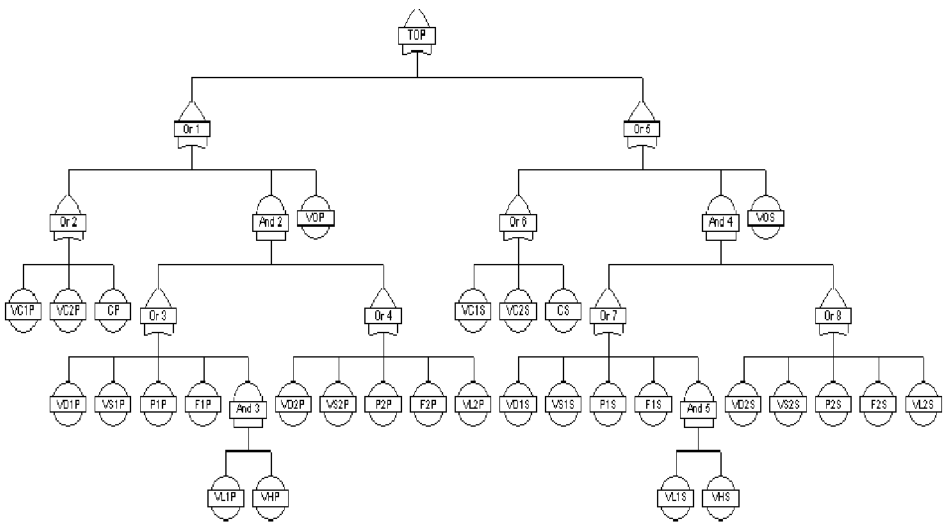

Fig. 2. Fault tree for the analyzed sea water cooling system for crossover valves in closed position
Rys. 2. Drzewo niezdatności dla analizowanego systemu chłodzenia wodą morską przy zaworach rozdzielających w położeniu zamkniętym

Besides, analyses were performed with exact *ERAC* calculation for a time dependent event model (exponential distribution). The operation of the engine room is according to the third DP class. This means that crossover valves between the engine rooms are closed, while the main electric buses are divided. Some failures at the beginning of the observation have been simulated, e.g. one strainer was clogged (F1P), two strainers were clogged – one in each engine room (F1P, F1S), one sea chest was out of operation (VL1P, VL2P). The results of the analysis of 4300 hours simulation are shown in Table 2 and Fig. 3 and 4.

Table 2

Unavailability characteristics of the analyzed system with some components in the down state at the start of operation

Charakterystyki niegotowości dla analizowanego systemu z niektórymi elementami niezdatnymi w chwili rozpoczęcia pracy

	Components that failed at the start of observation			
t	none	F1P	F1P, F1S	VL1P , VL2P
0	1.80E–04	5.70E–04	9.60E–04	2.70E–04
450	1.45E–01	3.27E–01	4.69E–01	3.84E–01
900	3.97E–01	5.91E–01	7.21E–01	6.88E–01
1350	6.16E–01	7.66E–01	8.54E–01	8.58E–01
1800	7.70E–01	8.70E–01	9.24E–01	9.40E–01
2250	8.68E–01	9.30E–01	9.61E–01	9.75E–01
2700	9.26E–01	9.62E–01	9.80E–01	9.90E–01
3150	9.59E–01	9.80E–01	9.89E–01	9.96E–01
3600	9.78E–01	9.89E–01	9.95E–01	9.99E–01
4050	9.88E–01	9.94E–01	9.97E–01	9.99E–01
4500	9.94E–01	9.97E–01	9.99E–01	1.00E+00

The presented characteristics show that with the higher number of failed components in the system, the unavailability function values are also higher.

Final conclusions

The presented methodology is applicable to coherent fault trees with binary logical operators. The operation of presented gates is independent of time. The static fault trees use these operators in combination with primary events represented by the constant probability of event occurrence.

If events or gates are time dependent, the built fault tree is called a dynamic fault tree. The group of time dependent logical operators i.e. spare gates (hot, warm, cold), priority *AND* gate, functional dependency gate etc. are not presented here.

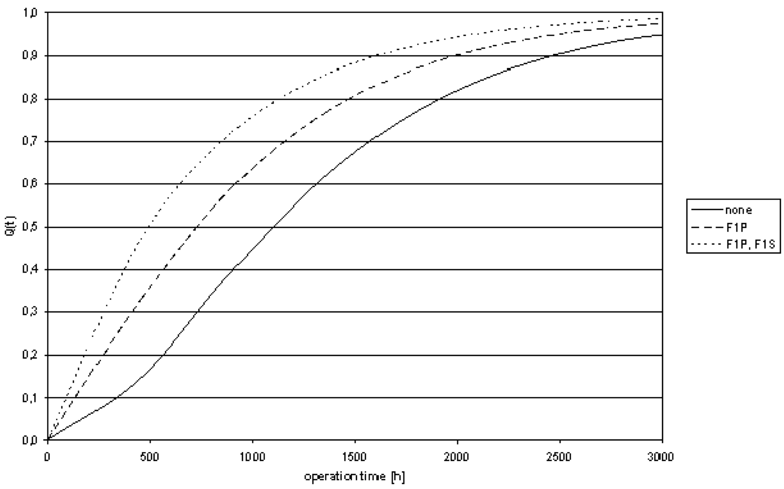

Fig. 3. Unavailability of the analyzed system for some strainers being out of operation at the start
Rys. 3. Niegotowość analizowanego systemu dla niektórych filtrów niezdatnych w chwili rozpoczęcia pracy

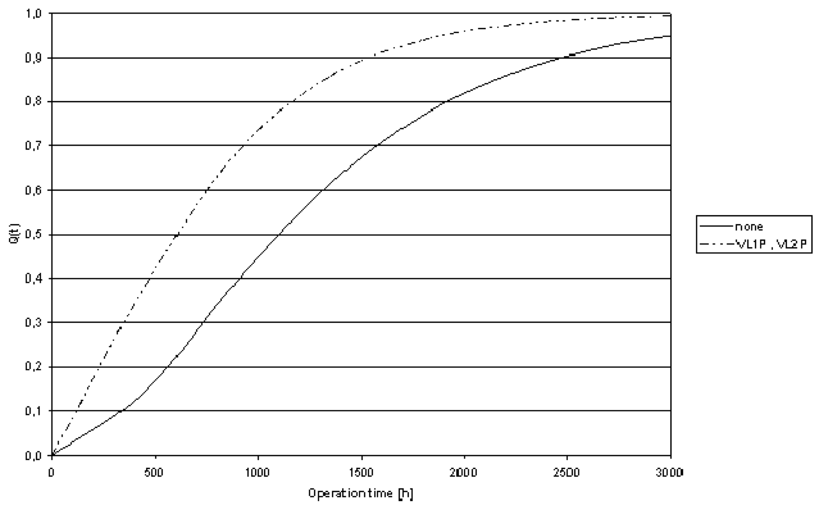

Fig. 4. Unavailability of the analyzed system for one sea chest being out of operation at the start
Rys. 4. Niegotowość analizowanego systemu dla jednego kosza ssawnego niezdatnego w chwili rozpoczęcia pracy

The application of time-dependent models offers a fuller description of the system behaviour during its operation than a classical model, which has been shown in Fig. 3 and 4. These have been preparedon the basis of previously presented values of reliability characteristics and the given fault tree model.

The presented method gives a convenient analysis of system dependability (e.g. unavailability) characteristics of the system at selected suitable values measures of the characterised events (faults in the technical system).

The classical binary model is very fast in computing, so it is often used in a preliminary reliability analysis.

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